GLOBAL
EDITION

Feedback Control of Dynamic Systems

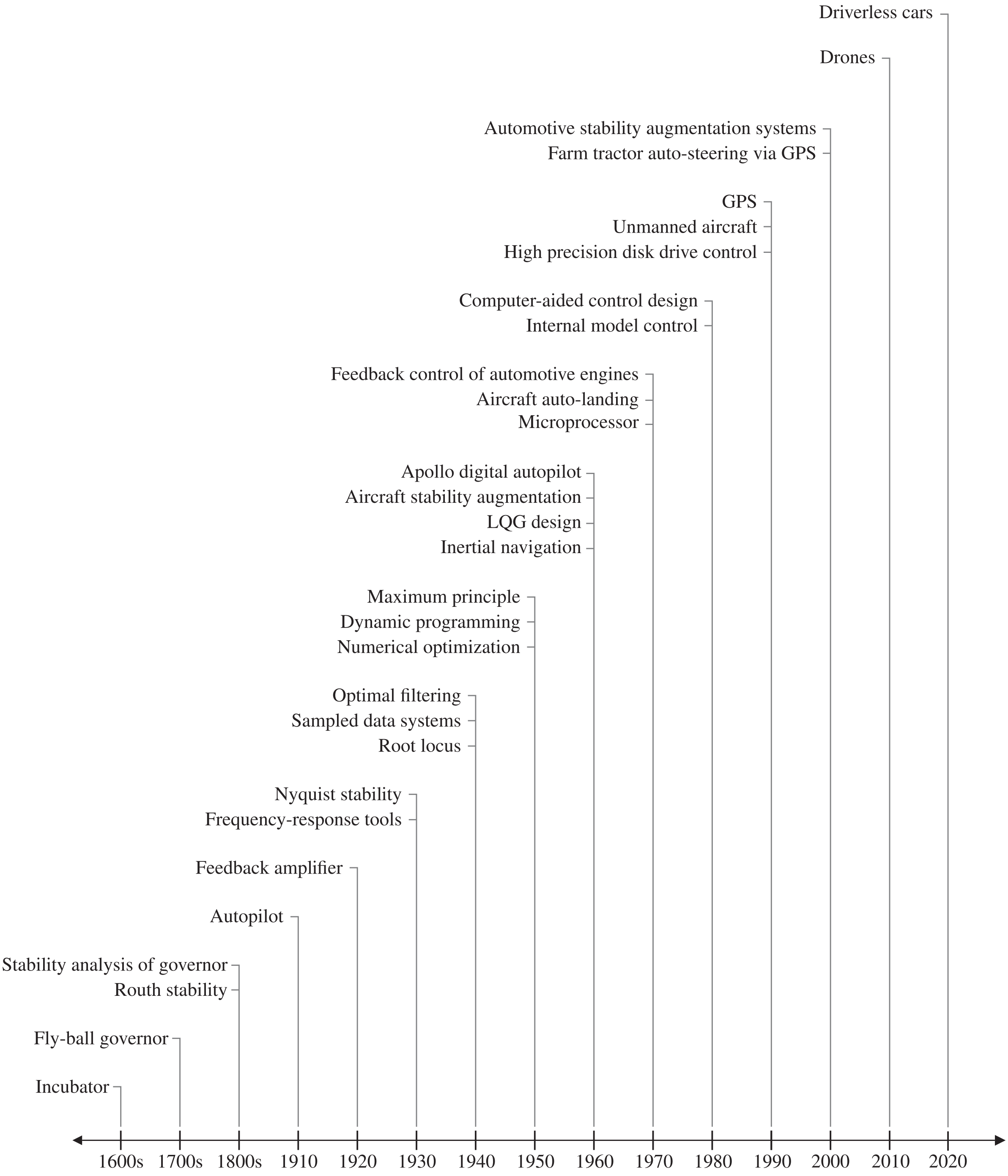
EIGHTH EDITION

Franklin • Powell • Emami-Naeini

Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	$1(t)$
3	$\frac{1}{s^2}$	t
4	$\frac{2!}{s^3}$	t^2
5	$\frac{3!}{s^4}$	t^3
6	$\frac{m!}{s^{m+1}}$	t^m
7	$\frac{1}{(s+a)}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt}-ae^{-at}$
17	$\frac{a}{(s^2+a^2)}$	$\sin at$
18	$\frac{s}{(s^2+a^2)}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s\left[(s+a)^2+b^2\right]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$

Chronological History of Feedback Control



Feedback Control of Dynamic Systems

Eighth Edition

Global Edition

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To Valerie, Daisy, Annika, Davenport, Malahat, Sheila, Nima, and to the memory of Gene

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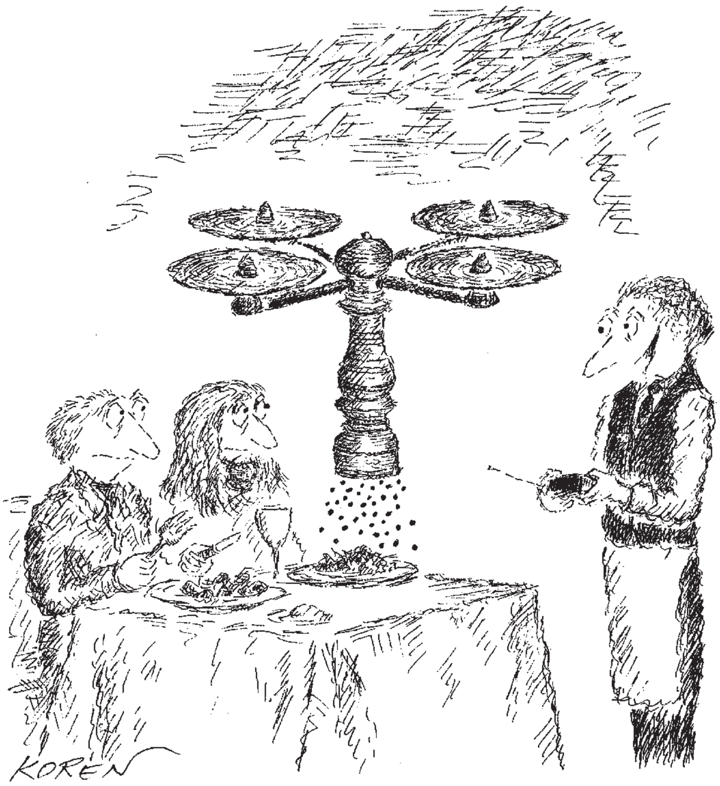
Preface

In this Eighth Edition we again present a text in support of a first course in control and have retained the best features of our earlier editions. For this edition, we have responded to a survey of users by adding some new material (for example, drone dynamics and control) and deleted other little-used material from the book. We have also updated the text throughout so that it uses the improved features of MATLAB®. Drones have been discussed extensively in the controls literature as well as the common press. They are being used in mining, construction, aerial photography, search and rescue, movie industry, package delivery, mapping, surveying, farming, animal research, hurricane hunting, and defense. Since feedback control is a necessary component of all the drones, we develop the equations of motion in Chapter 2, and follow that with control design examples in the chapters 5, 6, 7, and 10. They have great potential for many tasks and could speed up and lessen the cost of these activities. The figure below symbolizes the widespread interest in this exciting new field.



"Fresh pepper?"

Source: *Edward Koren/ The New Yorker © Conde Nast*

The basic structure of the book is unchanged and we continue to combine analysis with design using the three approaches of the root locus, frequency response, and state-variable equations. The text continues to include many carefully worked out examples to illustrate the material. As before, we provide a set of review questions at the end of each chapter with answers in the back of the book to assist the students in verifying that they have learned the material.

In the three central chapters on design methods we continue to expect the students to learn how to perform the very basic calculations by hand and make a rough sketch of a root locus or Bode plot as a sanity check on the computer results and as an aid to design. However, we introduce the use of Matlab early on in recognition of the universal use of software tools in control analysis and design. As before, we have prepared a collection of all the Matlab files (both "m"files and SIMULINK® "slx" files) used to produce the figures in the book. These are available along with the advanced material described above at our website at *www.pearsonglobaleditions.com*.

New to this Edition

We feel that this Eighth Edition presents the material with good pedagogical support, provides strong motivation for the study of control, and represents a solid foundation for meeting the educational challenges. We introduce the study of feedback control, both as a specialty of itself and as support for many other fields.

A more detailed list of the changes is:

- Deleted the disk drive and tape drive examples from Chapters 2, 7, and 10
- Added drone examples and/or problems in Chapters 2, 5, 6, 7, and 10
- Added a thermal system control example to Chapters 2 and 4
- Added a section on anti-windup for integral control in Chapter 9
- Added Cramer's Rule to chapter 2 and Appendix WB
- Updated Matlab commands throughout the book and in Appendix C
- Updated the section on PID tuning in chapter 4
- Updated the engine control and chemotaxis case studies in Chapter 10
- Over 60 of the problems in this edition are either new or revised from the 7th edition

Addressing the Educational Challenges

Some of the educational challenges facing students of feedback control are long-standing; others have emerged in recent years. Some of the challenges remain for students across their entire engineering education; others are unique to this relatively sophisticated course. Whether they are old or new, general or particular, the educational challenges we perceived were critical to the evolution of this text. Here, we will state several educational challenges and describe our approaches to each of them.

- CHALLENGE *Students must master design as well as analysis techniques.*

Design is central to all of engineering and especially so to control systems. Students find that design issues, with their corresponding opportunities to tackle practical applications, are particularly motivating. But students also find design problems difficult because design problem statements are usually poorly posed and lack unique solutions. Because of both its inherent importance and its motivational effect on students, design is emphasized throughout this text so confidence in solving design problems is developed from the start.

The emphasis on design begins in Chapter 4 following the development of modeling and dynamic response. The basic idea of feedback is introduced first, showing its influence on disturbance rejection, tracking accuracy, and robustness to parameter changes. The design orientation continues with uniform treatments of the root locus, frequency response, and state variable feedback techniques. All the treatments are aimed at providing the knowledge necessary to find a good feedback control design with no more complex mathematical development than is essential to clear understanding.

Throughout the text, examples are used to compare and contrast the design techniques afforded by the different design methods and, in the capstone case studies of Chapter 10, complex real-world design problems are attacked using all the methods in a unified way.

- CHALLENGE *New ideas continue to be introduced into control.*

Control is an active field of research and hence there is a steady influx of new concepts, ideas, and techniques. In time, some of these elements develop to the point where they join the list of things every control engineer must know. This text is devoted to supporting students equally in their need to grasp both traditional and more modern topics.

In each of our editions, we have tried to give equal importance to root locus, frequency response, and state-variable methods for design. In this edition, we continue to emphasize solid mastery of the underlying techniques, coupled with computer-based methods for detailed calculation. We also provide an early introduction to data sampling and discrete controllers in recognition of the major role played by digital controllers in our field. While this material can be skipped to save time without harm to the flow of the text, we feel that it is very important for students to understand that computer control is widely used and that the most basic techniques of computer control are easily mastered.

- CHALLENGE *Students need to manage a great deal of information.*

The vast array of systems to which feedback control is applied and the growing variety of techniques available for the solution of control problems means that today's student of feedback control must learn many new ideas. How do students keep their perspective as they plow through lengthy and complex textual passages? How do they identify highlights and draw appropriate conclusions? How do they review for exams? Helping students with these tasks was a criterion for the Fourth, Fifth, Sixth, and Seventh Editions and continues to be addressed in this Eighth Edition. We outline these features below.

FEATURE

1. *Chapter openers* offer perspective and overview. They place the specific chapter topic in the context of the discipline as a whole, and they briefly overview the chapter sections.
2. *Margin notes* help students scan for chapter highlights. They point to important definitions, equations, and concepts.
3. *Shaded highlights* identify key concepts within the running text. They also function to summarize important design procedures.
4. *Bulleted chapter summaries* help with student review and prioritization. These summaries briefly reiterate the key concepts and conclusions of the chapter.
5. *Synopsis of design aids*. Relationships used in design and throughout the book are collected inside the back cover for easy reference.
6. *The color blue* is used (1) to highlight useful pedagogical features, (2) to highlight components under particular scrutiny within block diagrams, (3) to distinguish curves on graphs, and (4) to lend a more realistic look to figures of physical systems.
7. *Review questions* at the end of each chapter with solutions in the back to guide the student in self-study
8. *Historical perspectives* at the end of each chapter provide some background and color on how or why the material in that particular chapter evolved.

- CHALLENGE *Students of feedback control come from a wide range of disciplines.*

Feedback control is an interdisciplinary field in that control is applied to systems in every conceivable area of engineering. Consequently, some schools have separate introductory courses for control within the standard disciplines and some, such as Stanford, have a single set of courses taken by students from many disciplines. However, to restrict the examples to one field is to miss much of the range and power of feedback but to cover the whole range of applications is overwhelming. In this book, we develop the interdisciplinary nature of the field and provide review material for several of the most common technologies so that students from many disciplines will be comfortable with the presentation. For Electrical Engineering students who typically have a good background in transform analysis, we include in Chapter 2 an introduction to writing equations of motion for mechanical mechanisms. For mechanical engineers, we include in Chapter 3 a review of the Laplace transform and dynamic response as needed in control. In addition, we introduce other technologies briefly and, from time to time, we present the equations of motion of a physical system without derivation but with enough physical description to be understood from a response point of view. Examples of some of the physical systems represented in the text include a quadrotor drone, a satellite tracking system, the fuel–air ratio in an automobile engine, and an airplane automatic pilot system.

Outline of the Book

The contents of the printed book are organized into ten chapters and three appendices. Optional sections of advanced or enrichment material marked with a triangle (Δ) are included at the end of some chapters. Examples and problems based on this material are also marked with a triangle (Δ). There are also four full appendices on the website plus numerous appendices that supplement the material in most of the chapters. The appendices in the printed book include Laplace transform tables, answers to the end-of-chapter review questions, and a list of Matlab commands. The appendices on the website include a review of complex variables, a review of matrix theory, some important results related to state-space design, and optional material supporting or extending several of the chapters.

In Chapter 1, the essential ideas of feedback and some of the key design issues are introduced. This chapter also contains a brief history of control, from the ancient beginnings of process control to flight control and electronic feedback amplifiers. It is hoped that this brief history will give a context for the field, introduce some of the key people who contributed to its development, and provide motivation to the student for the studies to come.

Chapter 2 is a short presentation of dynamic modeling and includes mechanical, electrical, electromechanical, fluid, and thermodynamic devices. This material can be omitted, used as the basis of review homework to smooth out the usual nonuniform preparation of students, or covered in-depth depending on the needs of the students.

Chapter 3 covers dynamic response as used in control. Again, much of this material may have been covered previously, especially by electrical engineering students. For many students, the correlation between pole locations and transient response and the effects of extra zeros and poles on dynamic response represent new material. Stability of dynamic systems is also introduced in this chapter. This material needs to be covered carefully.

Chapter 4 presents the basic equations and transfer functions of feedback along with the definitions of the sensitivity function. With these tools, open-loop and closed-loop control are compared with respect to disturbance rejection, tracking accuracy, and sensitivity to model errors. Classification of systems according to their ability to track polynomial reference signals or to reject polynomial disturbances is described with the concept of system type. Finally, the classical proportional, integral, and derivative (PID) control structure is introduced and the influence of the controller parameters on a system's characteristic equation is explored along with PID tuning methods.

Following the overview of feedback in Chapter 4, the core of the book presents the design methods based on root locus, frequency response, and state-variable feedback in Chapters 5, 6, and 7, respectively.

Chapter 8 develops the tools needed to design feedback control for implementation in a digital computer. However, for a complete treatment of feedback control using digital computers, the reader is referred to the companion text, *Digital Control of Dynamic Systems*, by Franklin, Powell, and Workman; Ellis-Kagle Press, 1998.

In Chapter 9, the nonlinear material includes techniques for the linearization of equations of motion, analysis of zero memory nonlinearity as a variable gain, frequency response as a describing function, the phase plane, Lyapunov stability theory, and the circle stability criterion.

In Chapter 10, the three primary approaches are integrated in several case studies, and a framework for design is described that includes a touch of the real-world context of practical control design.

Course Configurations

The material in this text can be covered flexibly. Most first-course students in controls will have some dynamics and Laplace transforms. Therefore, Chapter 2 and most of Chapter 3 would be a review for those students. In a ten-week quarter, it is possible to review Chapter 3, and cover all of Chapters 1, 4, 5, and 6. Most optional sections should be omitted. In the second quarter, Chapters 7 and 9 can be covered comfortably including the optional sections. Alternatively, some optional sections could be omitted and selected portions of Chapter 8 included. A semester course should comfortably accommodate Chapters 1–7, including the review materials of Chapters 2 and 3, if needed. If time remains after this core coverage, some introduction of digital control from Chapter 8, selected nonlinear issues from Chapter 9, and some of the case studies from Chapter 10 may be added.

The entire book can also be used for a three-quarter sequence of courses consisting of modeling and dynamic response (Chapters 2 and 3), classical control (Chapters 4–6), and modern control (Chapters 7–10).

Two basic 10-week courses are offered at Stanford and are taken by seniors and first-year graduate students who have not had a course in control, mostly in the departments of Aeronautics and Astronautics, Mechanical Engineering, and Electrical Engineering. The first course reviews Chapters 2 and 3 and covers Chapters 4–6. The more advanced course is intended for graduate students and reviews Chapters 4–6 and covers Chapters 7–10. This sequence complements a graduate course in linear systems and is the prerequisite to courses in digital control, nonlinear control, optimal control, flight control, and smart product design. Some of the subsequent courses include extensive laboratory experiments. Prerequisites for the course sequence include dynamics or circuit analysis and Laplace transforms.

Prerequisites to This Feedback Control Course

This book is for a first course at the senior level for all engineering majors. For the core topics in Chapters 4–7, prerequisite understanding of modeling and dynamic response is necessary. Many students will come into the course with sufficient background in those concepts from previous courses in physics, circuits, and dynamic response. For those needing review, Chapters 2 and 3 should fill in the gaps.

An elementary understanding of matrix algebra is necessary to understand the state-space material. While all students will have much of this in prerequisite math courses, a review of the basic relations is given in online Appendix WB and a brief treatment of particular material needed in control is given at the start of Chapter 7. The emphasis is on the relations between linear dynamic systems and linear algebra.

Supplements

The website *www.pearsonglobaleditions.com* includes the dot-m and dot-slx files used to generate all the Matlab figures in the book, and these may be copied and distributed to the students as desired. The websites also contain some more advanced material and appendices which are outlined in the Table of Contents. A Solutions Manual with complete solutions to all homework problems is available to instructors only.

Acknowledgments

Finally, we wish to acknowledge our great debt to all those who have contributed to the development of feedback control into the exciting field it is today and specifically to the considerable help and education we have received from our students and our colleagues. In particular, we have benefited in this effort by many discussions with the following who taught introductory control at Stanford: A. E. Bryson, Jr., R. H. Cannon, Jr., D. B. DeBra, S. Rock, S. Boyd, C. Tomlin, P. Enge, A. Okamura, and C. Gerdes. Other colleagues who have helped us include D. Fraser, N. C. Emami, B. Silver, M. Dorfman, K. Rudie, L. Pao, F. Khorrami, K. Lorell, M. Tischler, D. de Roover, R. Patrick, M. Berrios, J. K. Lee, J. L. Ebert, I. Kroo, K. Leung, and M. Schwager. Special thanks go to the many students who have provided almost all the solutions to the problems in the book.

We especially want to express our great appreciation for the contributions to the book by Gene Franklin. Gene was a mentor, teacher, advisor, and good friend to us both. We had many meetings as we collaborated on earlier editions of the book over the last 28 years of his life, and every single one of those meetings has been friendly and enjoyable as we meshed our views on how to present the material. We learned control along with humor from Gene in grad school classes, and we benefitted from his mentoring: in one case as a new assistant professor, and in the other as a Ph.D. advisee. Collectively, we collaborated on research, created new courses and laboratories, and written two textbooks over a period of 40 years. Gene always had a smile with a twinkle in his eye, and was a pleasure to work with; he was a true gentleman.

J.D.P.
A.E.-N.
Stanford, California

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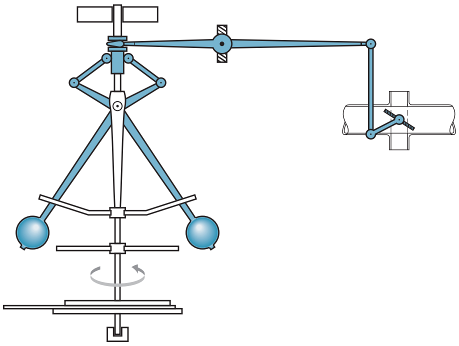
1

An Overview and Brief History of Feedback Control



A Perspective on Feedback Control

Feedback control of dynamic systems is a very old concept with many characteristics that have evolved over time. The central idea is that a dynamic system's output can be measured and fed back to a controller of some kind then used to affect the system. There are several variations on this theme.

A system that involves a person controlling a machine, as in driving an automobile, is called **manual** control. A system that involves machines only, as when room temperature can be set by a thermostat, is called **automatic** control. Systems designed to hold an output steady against unknown disturbances are called **regulators**, while systems designed to track a reference signal are called **tracking** or **servo** systems. Control systems are also classified according to the information used to compute the controlling action. If the controller does *not* use a measure of the system output being controlled in computing the control action to take, the system is called **open-loop** control. If the controlled output signal *is* measured and fed back for use in the control computation, the system is called **closed-loop** or **feedback** control. There are many other important properties of control systems in addition to these most basic characteristics. For example, we will mainly consider feedback of current measurements as opposed to predictions of the future; however, a very familiar example illustrates the limitation imposed by that assumption. When driving a car, the use of simple feedback corresponds to driving in a thick fog where one can *only see the road immediately at the front of the car* and is unable to see the future required position! Looking at the road ahead is a form of predictive control and this information, which has obvious advantages, would always be used where it is available. In most automatic control situations studied in this book, observation of the future track or disturbance is not possible. In any case, the control designer should study the process to see if any information could anticipate either a track to be followed or a disturbance to be rejected. If such a possibility is feasible, the control designer should use it to **feedforward** an early warning to the control system. An example of this is in the control of steam pressure in the boiler of an electric power generation plant. The electricity demand cycle over a day is well known; therefore, when it is known that there will soon be an increased need for electrical power, that information can be fed forward to the boiler controller in anticipation of a soon-to-be-demanded increase in steam flow.

The applications of feedback control have never been more exciting than they are today. Feedback control is an essential element in aircraft of all types: most manned aircraft, and all unmanned aircraft from large military aircraft to small drones. The FAA has predicted that the number of drones registered in the U.S. will reach 7 million by 2020! Automatic landing and collision avoidance systems in airliners are now being used routinely, and the use of satellite navigation in future designs promises a revolution in our ability to navigate aircraft in an ever more crowded airspace. The use of feedback control in driverless cars is an essential element to their success. They are now under extensive development, and predictions have been made that driverless cars will ultimately reduce the number of cars on the road by a very large percentage. The use of feedback control in surgical robotic systems is also emerging. Control is essential to the operation of systems from cell phones to jumbo jets and from washing machines to oil refineries as large as a small city. The list goes on and on. In fact, many engineers refer to control as a *hidden technology* because of its essential importance to so many devices and systems while being mainly out of sight. The future will no doubt see engineers create even more imaginative applications of feedback control.

Chapter Overview

In this chapter, we begin our exploration of feedback control using a simple familiar example: a household furnace controlled by a thermostat. The generic components of a control system are identified within the context of this example. In another example in Section 1.2—an automobile cruise control—we will develop the elementary static equations and assign numerical values to elements of the system model in order to compare the performance of open-loop control to that of feedback control when dynamics are ignored. Section 1.3 then introduces the key elements in control system design. In order to provide a context for our studies, and to give you a glimpse of how the field has evolved, Section 1.4 provides a brief history of control theory and design. In addition, later chapters have brief sections of additional historical notes on the topics covered there. Finally, Section 1.5 provides a brief overview of the contents and organization of the entire book.

1.1 A Simple Feedback System

In feedback systems, the variable being controlled—such as temperature or speed—is measured by a sensor and the measured information is fed back to the controller to influence the controlled variable. The principle is readily illustrated by a very common system, the household furnace controlled by a thermostat. The components of this system and their interconnections are shown in Fig. 1.1. Such an illustration identifies the major parts of the system and shows the directions of information flow from one component to another.

We can easily analyze the operation of this system qualitatively from the graph. Suppose both the temperature in the room where the thermostat is located and the outside temperature are significantly below the reference temperature (also called the setpoint) when power is applied. The thermostat will be *on* and the control logic will open the furnace gas valve and light the fire box. This will cause heat Q_{in} to be supplied to the house at a rate that will be significantly larger than the heat loss Q_{out}. As a result, the room temperature will rise until it exceeds the thermostat reference setting by a small amount. At this time, the furnace will be turned off and the room temperature will start to fall toward the outside value. When it falls a small amount below the setpoint,[1] the thermostat will come on again and the cycle will repeat. Typical plots of room temperature along with the furnace cycles of on and off are shown in Fig. 1.1. The outside temperature remains at 50°F and the thermostat is initially set at 55°F. At 6 a.m., the thermostat is stepped to 65°F and the furnace brings it to that level and cycles the temperature around that value thereafter. Notice the house is well insulated, so the fall of temperature with the furnace off is significantly slower than the rise with the furnace on. From this example, we can identify the generic components of the elementary feedback control system, as shown in Fig. 1.2.

The central component of this feedback system is the **process** whose output is to be controlled. In our example the process would be the house whose output is the room temperature and the **disturbance** to

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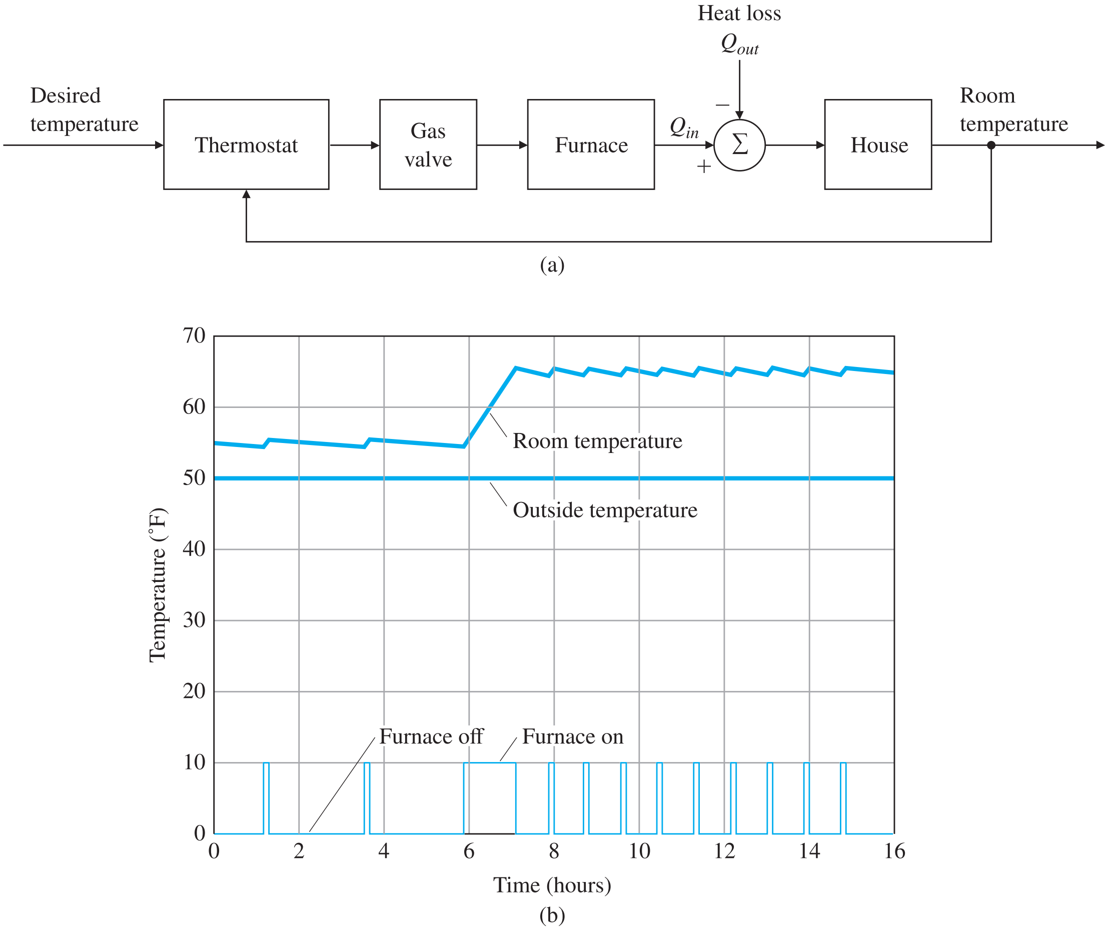
[1] The **setpoint**, **reference**, and **desired input** are all the same thing and shown in Figs. 1.1–1.3.



(a)

(b)

Figure 1.1

Feedback control: (a) component block diagram of a room temperature control system; (b) plot of room temperature and furnace action

the process is the flow of heat from the house, Q_{out}, due to conduction through the walls and roof to the lower outside temperature. (The outward flow of heat also depends on other factors such as wind, open doors, and so on.) The design of the process can obviously have a major impact on the effectiveness of the controls. The temperature of a well-insulated house with thermopane windows is clearly easier to control than otherwise. Similarly, the design of aircraft with control in mind makes a world of difference to the final performance. In every case, the earlier the concepts of control are introduced into the process design, the better. The **actuator** is the device that can influence the controlled variable of the process. In our case, the actuator is a gas furnace. Actually, the furnace usually has a pilot light or striking mechanism, a gas valve, and a blower fan, which turns on or off depending on the air temperature in the furnace. These details illustrate the fact that many feedback systems contain components that themselves

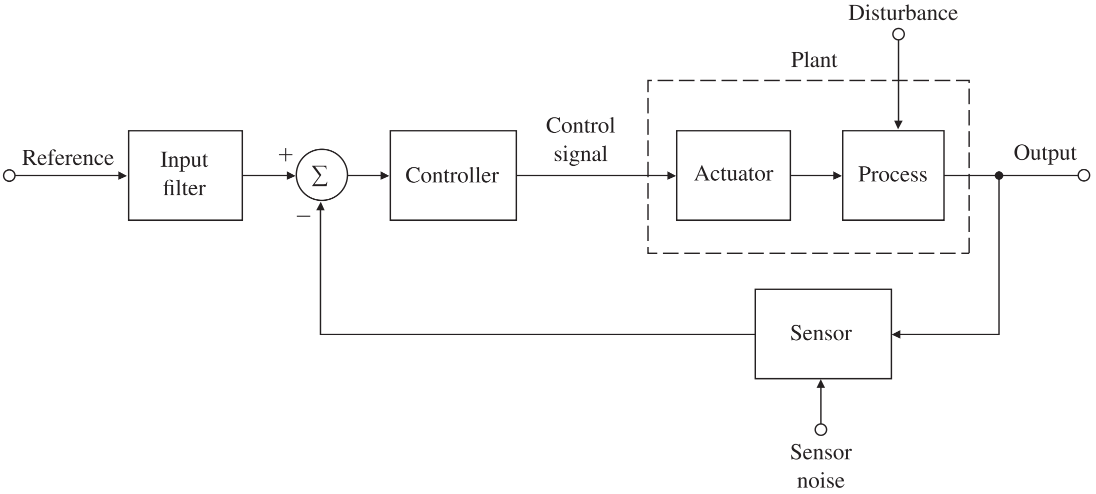


Figure 1.2
Component block diagram of an elementary feedback control

form other feedback systems.[2] The central issue with the actuator is its ability to move the process output with adequate speed and range. The furnace must produce more heat than the house loses on the worst day, and must distribute it quickly if the house temperature is to be kept in a narrow range. Power, speed, and reliability are usually more important than accuracy. Generally, the process and the actuator are intimately connected and the control design centers on finding a suitable input or control signal to send to the actuator. The combination of process and actuator is called the **plant**, and the component that actually computes the desired control signal is the **controller**. Because of the flexibility of electrical signal processing, the controller typically works on electrical signals, although the use of pneumatic controllers based on compressed air has a long and important place in process control. With the development of digital technology, cost-effectiveness and flexibility have led to the use of digital signal processors as the controller in an increasing number of cases. The component labeled **thermostat** in Fig. 1.1 measures the room temperature and is called the **sensor** in Fig. 1.2, a device whose output inevitably contains sensor noise. Sensor selection and placement are very important in control design, for it is sometimes not possible for the true controlled variable and the sensed variable to be the same. For example, although we may really wish to control the house temperature as a whole, the thermostat is in one particular room, which may or may not be at the same temperature as the rest of the house. For instance, if the thermostat is set to 68°F but is placed in the living room near a roaring fireplace, a person working in

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[2] Jonathan Swift (1733) said it this way: “So, Naturalists observe, a flea Hath smaller fleas that on him prey; And these have smaller still to bite ‘em; And so proceed, *ad infinitum*.” Swift, J., On Poetry: A Rhapsody, 1733, J. Bartlett, ed., Familiar Quotations, 15th ed., Boston: Little Brown, 1980.

the study could still feel uncomfortably cold.[3,4] As we will see, in addition to placement, important properties of a sensor are the accuracy of the measurements as well as low noise, reliability, and linearity. The sensor will typically convert the physical variable into an electrical signal for use by the controller. Our general system also includes an **input filter** whose role is to convert the reference signal to electrical form for later manipulation by the controller. In some cases, the input filter can modify the reference command input in ways that improve the system response. Finally, there is a **controller** to compute the difference between the reference signal and the sensor output to give the controller a measure of the system error. The thermostat on the wall includes the sensor, input filter, and the controller. A few decades ago, the user simply set the thermostat manually to achieve the desired room temperature at the thermostat location. Over the last few decades, the addition of a small computer in the thermostat has enabled storing the desired temperature over the day and week and more recently, thermostats have gained the ability to learn what the desired temperature should be and to base that value, in part, on whether anybody will be home soon! A thermostat system that includes a motion detector can determine whether anybody is home and learns from the patterns observed what the desired temperature profile should be. The process of learning the desired setpoint is an example of artificial intelligence (AI) or machine learning, which is gaining acceptance in many fields as the power and affordability of computers improve. The combination of feedback control, AI, sensor fusion, and logic to tie it all together will become an essential feature in many future devices such as drones, driverless cars, and many others.

This text will present methods for analyzing feedback control systems and will describe the most important design techniques engineers can use in applying feedback to solve control problems. We will also study the specific advantages of feedback that compensate for the additional complexity it demands.

1.2 A First Analysis of Feedback

The value of feedback can be readily demonstrated by quantitative analysis of a simplified model of a familiar system, the cruise control of an automobile (see Fig. 1.3). To study this situation analytically, we

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[3] In the renovations of the kitchen in the house of one of the authors, the new ovens were placed against the wall where the thermostat was mounted on the other side. Now when dinner is baked in the kitchen on a cold day, the author freezes in his study unless the thermostat is reset.

[4] The story is told of the new employee at the nitroglycerin factory who was to control the temperature of a critical part of the process manually. He was told to “keep that reading below 300°.” On a routine inspection tour, the supervisor realized that the batch was dangerously hot and found the worker holding the thermometer under cold water tap to bring it down to 300°. They got out just before the explosion. Moral: sometimes automatic control is better than manual.

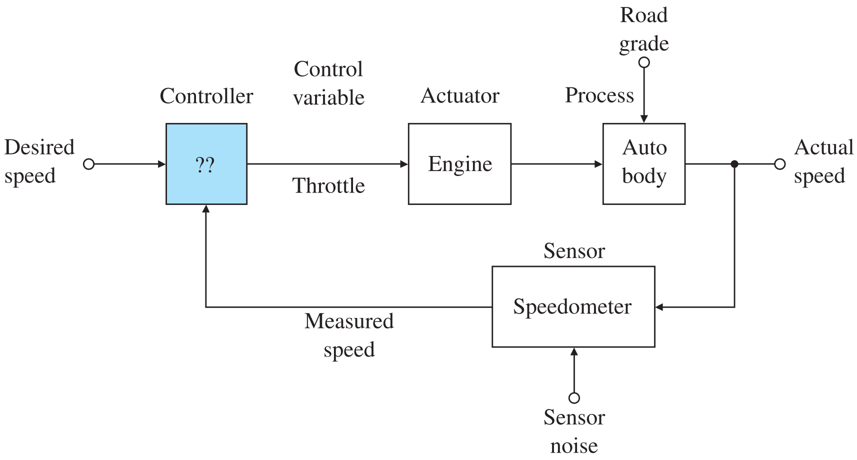


Figure 1.3
Component block diagram of automobile cruise control

need a mathematical **model** of our system in the form of a set of quantitative relationships among the variables. For this example, we ignore the dynamic response of the car and consider only the steady behavior. (Dynamics will, of course, play a major role in later chapters.) Furthermore, we assume that for the range of speeds to be used by the system, we can approximate the relations as linear. After measuring the speed of the vehicle on a level road at 65 mph, we find that a 1° change in the throttle angle (our control variable, u) causes a 10 mph change in speed (the output variable, y), hence the value 10 in the box between u and y in Fig. 1.4, which is a **block diagram** of the plant. Generally, the block diagram shows the mathematical relationships of a system in graphical form. From observations while driving up and down hills, it is found that when the grade changes by 1%, we measure a speed change of 5 mph, hence the value 0.5 in the upper box in Fig. 1.4, which reflects that a 1% grade change has half the effect of a 1° change in the throttle angle. The speedometer is found to be accurate to a fraction of 1 mph and will be considered exact. In the block diagram, the connecting lines carry signals and a block is like an ideal amplifier which multiplies the signal at its input by the value marked in the block to give the output signal. To sum two or more signals, we show lines for the signals coming into a **summer**, a circle with the summation sign Σ inside. An algebraic sign (plus or minus) beside each arrow head indicates whether the input

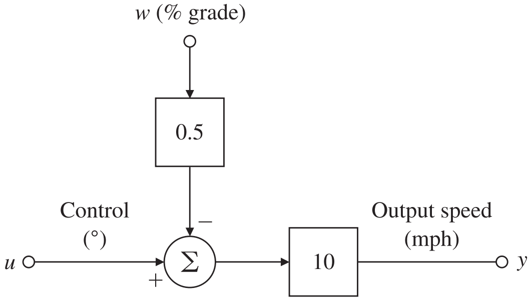
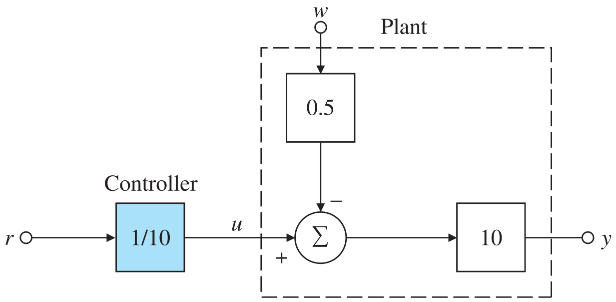


Figure 1.4
Block diagram of the cruise control plant

Figure 1.5

Open-loop cruise control



adds to or subtracts from the total output of the summer. For this analysis, we wish to compare the effects of a 1% grade on the output speed when the reference speed is set for 65 with and without feedback to the controller.

Open-loop control

In the first case, shown in Fig. 1.5, the controller does not use the speedometer reading but sets $u = r/10$, where r is the reference speed, which is, 65 mph. This is an example of an **open-loop control system.** The term *open-loop* refers to the fact that there is no closed path or loop around which the signals go in the block diagram; that is, the control variable u is independent of the output variable, y. In our simple example, the open-loop output speed, y_{ol}, is given by the equations

$$\begin{aligned} y_{ol} &= 10(u - 0.5w) \\ &= 10\left(\frac{r}{10} - 0.5w\right) \\ &= r - 5w. \end{aligned}$$

The error in output speed is

$$\begin{aligned} e_{ol} &= r - y_{ol} \qquad (1.1) \\ &= 5w, \qquad (1.2) \end{aligned}$$

and the percent error is

$$\%\,error = 500\frac{w}{r}. \qquad (1.3)$$

If $r = 65$ and the road is level, then $w = 0$ and the speed will be 65 with no error. However, if $w = 1$ corresponding to a 1% grade, then the speed will be 60 and we have a 5-mph error, which is a 7.69% error in the speed. For a grade of 2%, the speed error would be 10 mph, which is an error of 15.38%, and so on. The example shows that there would be no error when $w = 0$, but this result depends on the controller gain being the exact inverse of the plant gain of 10. In practice, the plant gain is subject to change and if it does, errors are introduced by this means also. If there is an error in the plant gain in open-loop control, the percent speed error would be the same as the percent plant-gain error.

The block diagram of a feedback scheme is shown in Fig. 1.6, where the controller gain has been set to 10. In this simple example, we have assumed that we have an ideal sensor providing a measurement of y_{cl}. In this case, the equations are